Nuclear Theory - Course 127

EFFECT OF ENRICHMENT, FUEL ARRANGEMENT AND FUEL BURNUP ON THE FOUR FACTOR FORMULA

When considering the function and properties of the moderator, it was stated that a moderator has to be provided, in addition to fuel, in a reactor in order to slow down the neutrons to thermal energies.

A reactor containing fuel alone can not maintain a chain reaction unless the fuel is highly enriched and it operates as a fast reactor. However, it was not specified how the fuel and moderator would be arranged. There are two possible arrangements:

(a) A homogeneous system, in which the fuel and moderator are intimately mixed together. The uranium would either be in solution in the moderator or in a fine suspension called a slurry.

(b) A heterogeneous system, in which the fuel is in the form of lumps or rods arranged in a regular manner in the moderator.

The effect of such arrangements on the four factor formula, as well as the effect of enrichment and fuel burnup, will now be considered.

Homogeneous System

Suppose that the fuel and moderator are intimately mixed. When the fission neutron is born, it would be in contact with the moderator immediately and would collide with several moderator nuclei before encountering another fuel nucleus. The neutron would not therefore have enough energy to cause fast fissions in U-238 and $\epsilon = 1$. What of the other factors?

The number of neutrons, η , produced for each thermal neutron captured in the fuel will depend only on the fuel composition. If ν fast neutrons are produced at each U-235 fission, then:

$$\eta = \frac{\nu N_5 \sigma_f}{N_5\sigma_5 + N_8\sigma_8} = \frac{\nu \sigma_f R}{R\sigma_5 + \sigma_8} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where N_5 and N_8 are the numbers of U-235 and U-238 atoms, respectively, per unit volume of fuel, σ_f is the thermal neutron fission cross section of U-235, σ_5 and σ_8 are the absorption cross sections of U-235 and U-238, respectively, and R is the ratio N_5/N_8.

For natural uranium

$$\begin{aligned} R &= 0.715\% = 0.00715 \\ \nu &= 2.48 \\ \sigma_f &= 580 \text{ barns} \\ \sigma_5 &= 700 \text{ barns} \\ \sigma_8 &= 2.8 \text{ barns} \end{aligned}$$

$$\text{Hence } \eta = \frac{2.48 \times 580 \times 0.00715}{(0.00715 \times 700) + 2.8} = 1.32$$

$$\text{Now } k_\infty = \eta \epsilon pf = 1 \text{ for criticality}$$

$$\text{or } pf = \frac{1}{1.32} = 0.77$$

Thus, the product pf must be at least equal to 0.77 if such a homogeneous system is to be critical and a chain reaction be maintained. Is this possible with natural uranium fuel? If natural uranium fuel is to be used, the only way to vary p and f is to vary the ratio of moderator to fuel atoms in the homogeneous mixture. Table I below shows how p, f and the product pf vary as the ratio of moderator to fuel atoms is changed in a homogeneous mixture of graphite and natural uranium. The value of k_∞ is also given to show that it is less than 1, which is the minimum value, ignoring leakage, required for criticality.

TABLE I

nat. U + graphite

Mod. Atoms / Fuel Atoms	p	f	pf	k_∞
200	0.579	0.889	0.515	0.68
300	0.643	0.842	0.541	0.71
400	0.682	0.800	0.546	0.72
500	0.693	0.762	0.528	0.70

From the table it may be seen that the problem is that as the moderator/fuel ratio is increased, p increases, because there is better thermalization of neutrons, but f decreases, because there is more neutron capture in the moderator and less neutron capture in the fuel. The maximum value pf can have is only 0.55, whereas the value must be 0.77 to sustain a chain reaction. So a chain reaction cannot be maintained with a homogeneous mixture of graphite and natural uranium.

A similar situation exists with homogeneous systems of natural uranium and either light water or beryllium. With heavy water as moderator, the optimum ratio of moderator to fuel gives a maximum value of pf of 0.78, which corresponds to a maximum value of k = 1.03. This does not allow for neutron leakage nor does it allow for buildup of fission product poison. This means that the size of reactor required, with a natural uranium - heavy water homogeneous mixture, would be too large to be practical.

The Effect of Fuel Enrichment

What can be done to make such a system practical? One answer lies in increasing the value of η. For a practical reactor, in which fuel consumption and poison accumulation are allowed for, k would have to be around 1.1 to 1.2. Therefore the value of η required is given by:

$$\eta = \frac{1.2}{pf} = \frac{1.2}{0.78} = 1.54 \text{ for } D_2O \text{ moderator}$$

$$\eta = \frac{1.2}{0.55} = 2.18 \text{ for graphite moderator}$$

The only way to increase η is to increase the U-235 concentration (R) in the fuel, ie, enrich the fuel. Fig. 1 shows how the value of η varies with the U-235 concentration in the fuel. The U-235 concentration in natural uranium is 0.00715 (0.715%). For a homogeneous D_2O moderated system to be practical, the U-235 concentration would have to be increased to about 0.02 in order to obtain a value of 1.54 for η. Thus the fuel would contain 2% U-235. It would appear from the graph that the required value of η of 2.18 for a graphite moderated system, cannot be attained. However, enrichment of fuel decreases the U-238 content and increases p. Thus, the value of η, required for a homogeneous

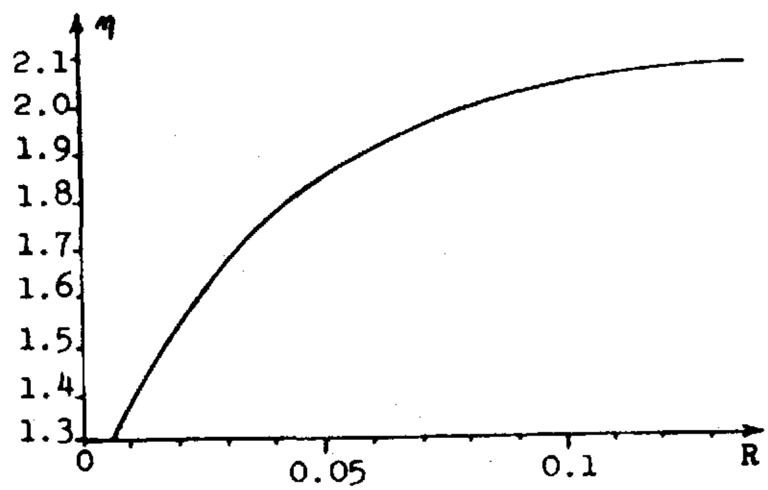

Fig. 1

mixture of graphite and fuel, decreases so that it can be attained with an enrichment of about 20% U-235.

It would also seem, from the graph, that there is little to be gained in the value of η by making R greater than 0.1. However, further enrichment does increase the value of p because of the decrease in the U-238 content.

Heterogeneous System

Even though enrichment does enable a homogeneous system to go critical, practical difficulties with such a system still exist. Refuelling problems, transportation of the heat released, fission product contamination, and criticality problems outside the reactor, are some of the difficulties encountered. Then, of course, there is also the very high capital cost of an enrichment plant and the obvious advantages to be gained using natural uranium, which requires only chemical processing. The alternative is to use a heterogeneous system if this is feasible. In such a system the fuel is located in discrete lumps in the moderator, as shown in Fig. 2, so that each fuel lump is surrounded by moderator material.

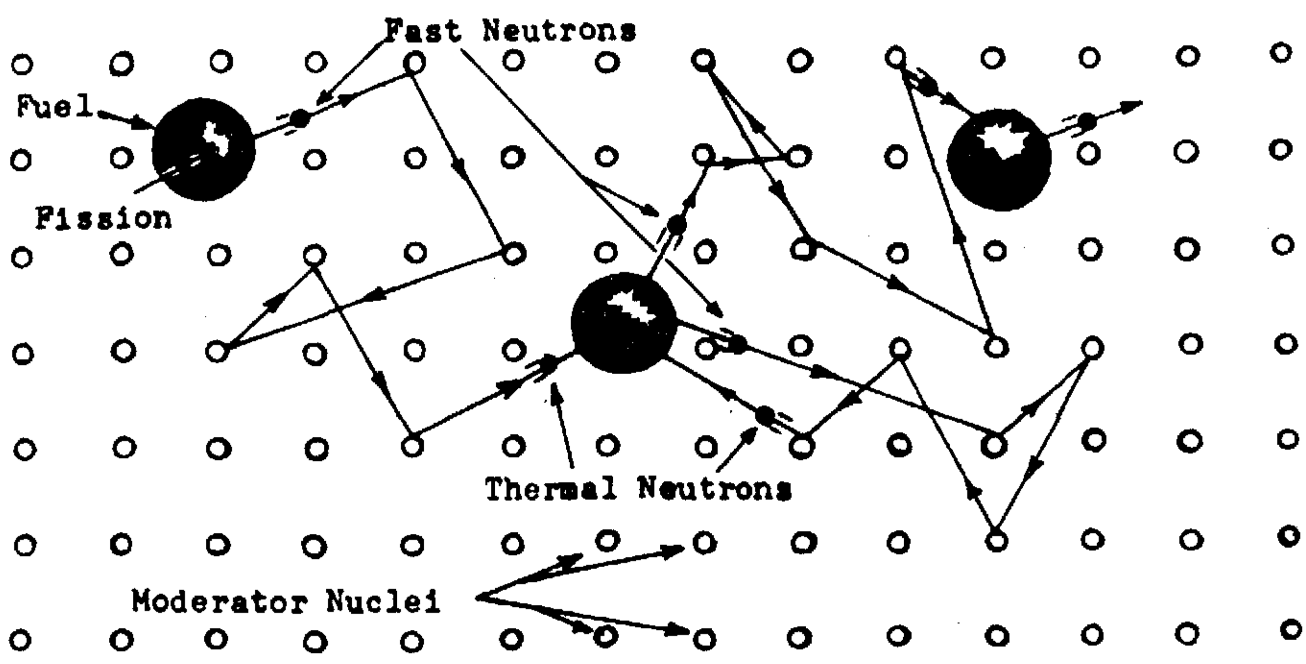

Fig. 2

Fissions then occur in the fuel lumps and the neutrons are partially slowed down by inelastic scattering as they escape from the fuel. The slowing down process is completed by elastic scattering collisions with moderator nuclei before the neutrons enter other lumps of fuel. The separation between fuel lumps will determine how many elastic scattering collisions occur and, therefore, whether the neutron with average energy of 2 Mev is thermalized or whether neutrons of all energies are thermalized.

In practice, the fuel is in the form of rods or fuel ELEMENTS arranged in bundles, the rods being arranged in a regular pattern or LATTICE in the moderator, as shown in Fig. 3.

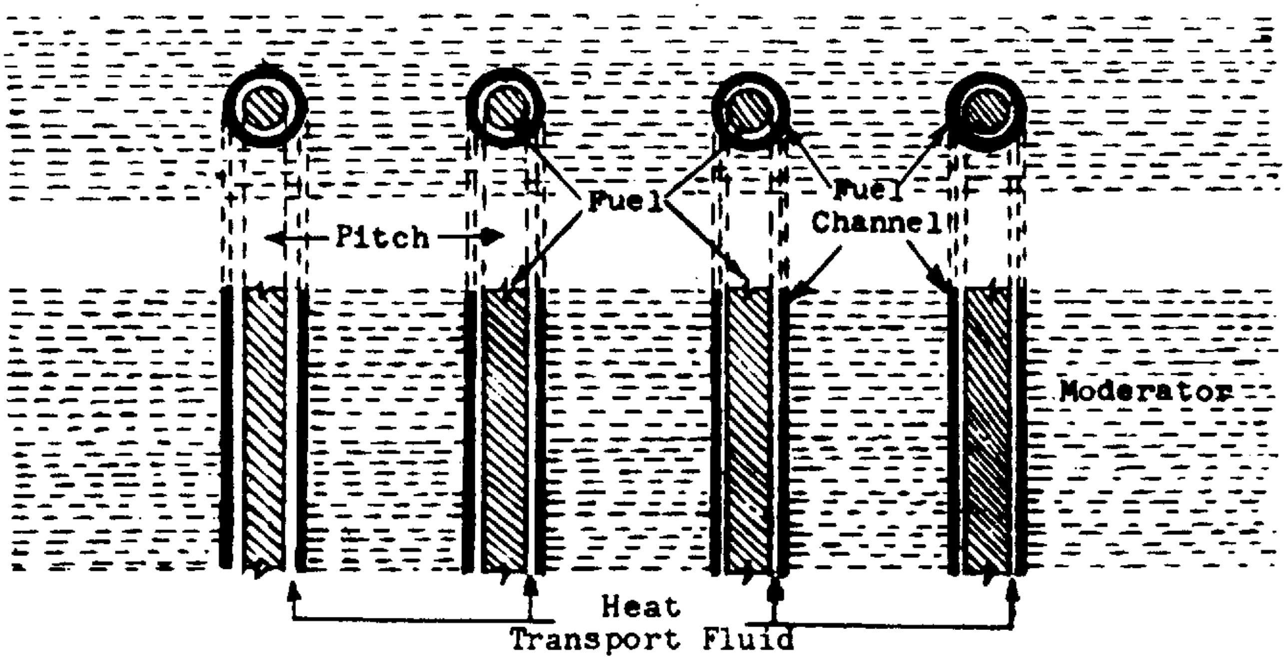

Fig. 3

This arrangement of rods inside tubes, or fuel channels, has several advantages:

(a) The location of the fuel is fixed by the fuel channel.

(b) Replacement of spent fuel by new fuel is easier.

(c) Heat can be removed from the fuel by passing the heat transport fluid along the fuel channel.

What nuclear advantages are to be gained by such an arrangement? The advantages, if any, will depend on what changes occur in the factors ϵ, η, p and f.

(a) The fast fission factor ϵ is increased. The fast fission neutrons are now released in the fuel rods and are, therefore, more likely to cause fast fissions in U-238 before escaping from the rod into the moderator. The value of ϵ will be between 1.02 and 1.03 instead of 1.0.

(b) The value of p, the resonance escape probability, is increased. Some slowing down by inelastic scattering occurs in the fuel. Also, if the fuel rods are placed at a distance apart, comparable to the average distance travelled by the neutron while slowing down, then the neutron will stand a good chance of becoming thermalized before entering the next fuel rod. Values of p of around 0.9 can be obtained by such an arrangement, which will undoubtedly increase pf sufficiently to enable a chain reaction to be maintained without enrichment.

There is a limit to the lattice PITCH, or rod separation. If the pitch is increased so that all the neutrons are thermalized, the value of p increases but the neutron capture increases. If the separation or pitch is too great, so that the neutrons are thermalized well before they enter a fuel lump, the reactor is said to be overmoderated. If the rods are too close, the reactor is undermoderated. With a moderator such as heavy water, which has a low capture cross section, there is a tendency to overmoderate in order to make p as large as possible without substantially decreasing f.

(c) The thermal utilization factor f is decreased. In a heterogeneous system, the average thermal neutron flux in the fuel is lower than it is in the moderator. There is, therefore, a tendency for relatively fewer neutrons to be captured in the fuel than there is in a homogeneous system. However, the decrease in f is small and, with heavy water moderator, the value of f is still higher than 0.9.

(d) The value of η is unchanged by using a heterogeneous system.

The overall effect is to increase the value of k sufficiently to make the chain reaction possible, without enrichment. The maximum possible value of k_∞ is now greater than that given by:

$$k_\infty = 1.03 \times 0.9 \times 0.9 \times 1.32 = 1.102$$

<u>Example</u>:

For the NPD equilibrium core:

$\eta = 1.229$; $\epsilon = 1.021$; $p = 0.918$; $f = 0.933$

Therefore $k_\infty = 1.075$

For Douglas Point:

$\eta = 1.1757$; $\epsilon = 1.0267$; $p = 0.8995$; $f = 0.9503$

Therefore $k_{\infty} = 1.032$

There are some significant differences between the two sets of parameters:

(a) The Douglas Point reactor is larger than the NPD reactor and a smaller allowance is necessary for neutron leakage. Hence, the value of k_{∞} is smaller for Douglas Point than for NPD.

(b) The lattice pitch for Douglas Point is only 9" compared with 10-1/4" for NPD. This accounts for the smaller value of p and the larger value of f for the Douglas Point reactor.

(c) The values are given for fuel from which U-235 has been removed by fission and in which some buildup of Pu-239 has occurred. The difference in the values of η and ϵ are thus due to a difference in burnup of the fuel in NPD and Douglas Point.

The degree of fuel subdivision will also have some effect on ϵ. The factor ϵ will increase with rod radius, since the longer the fission neutron remains in the fuel the more likely fast fissions are to occur. The fact that all the Douglas Point fuel is 19-element fuel, whereas the NPD fuel is a mixture of 7-element and 19-element fuel, will tend to make ϵ for NPD higher than for Douglas Point. However, this is completely masked by the effect of fuel burnup.

Effect of Fuel Burnup

As U-235 fissions occur in the fuel, the U-235 is being used up. However, Plutonium-239 is being produced from U-238 and Pu-241 is being produced by neutron capture in Pu-239 and then in Pu-240. Both Pu-239 and Pu-241 are fissionable with thermal neutrons. So, initially at least, we have fissionable U-235 burnt and fissionable Pu-239 and Pu-241 being produced. Later Pu-239 and Pu-241 will be burnt as well as produced and, eventually, all the U-235 will be used up and Plutonium alone will be burnt.

The burnup of U-235 and the production of Plutonium does not affect either p or ϵ to any great extent. However, the capture and fission cross sections of Pu-239 are substantially greater than those of U-235. This means that, although the

Plutonium is not produced as fast as U-235 is burnt up, there is initially an increase in the product ηf and, therefore, in the multiplication factor k. However, later, the burnup of Plutonium causes k to start to decrease even if poisons are ignored. Before the change in k with burnup can be shown, a method must be established of measuring fuel burnup. The possible definitions of burnup are as follows:

(a) Burnup is the percentage of the original fissile atoms burnt.

(b) Burnup is the percentage of the total fuel atoms burnt.

(c) Burnup is the heat extracted (in Megawatt days) per tonne (10^6 gms) of fuel.

Burnup, then, may be measured as a percentage of U-235 atoms burnt. If all the U-235 is used up, the burnup is 100%. Since Plutonium is being produced and burnt as well as all the U-235, the burnup can be greater than 100%.

If the U-235 forms a fraction E of the total fuel atoms,

$$\begin{aligned} b\% \text{ of U-235 burnup} &= Eb\% \text{ total fuel burnup} \\ &= 10{,}000\ Eb \text{ Megawatt-days/tonne} \end{aligned}$$

Therefore, if a reactor core was big enough to give a burnup of 10,000 Mwd/tonne, then Eb = 1 or 1% of all fuel atoms are burnt. If E = 0.00715, as it is in natural uranium,

$$b = \frac{1}{0.00715} = 140\% \text{ of U-235 atoms}$$

That is, the total number of U-235, Pu-239 and Pu-241 atoms fissioned is equal to 140% of the original amount of U-235 present.

A typical curve, showing how k varies with burnup, is shown in Fig. 4. No allowance has been made for fission product poison buildup which, initially, tends to mask out the Plutonium buildup. The increase in reactivity, during the early fuel irradiation, does occur, however, and an allowance must be made for it.

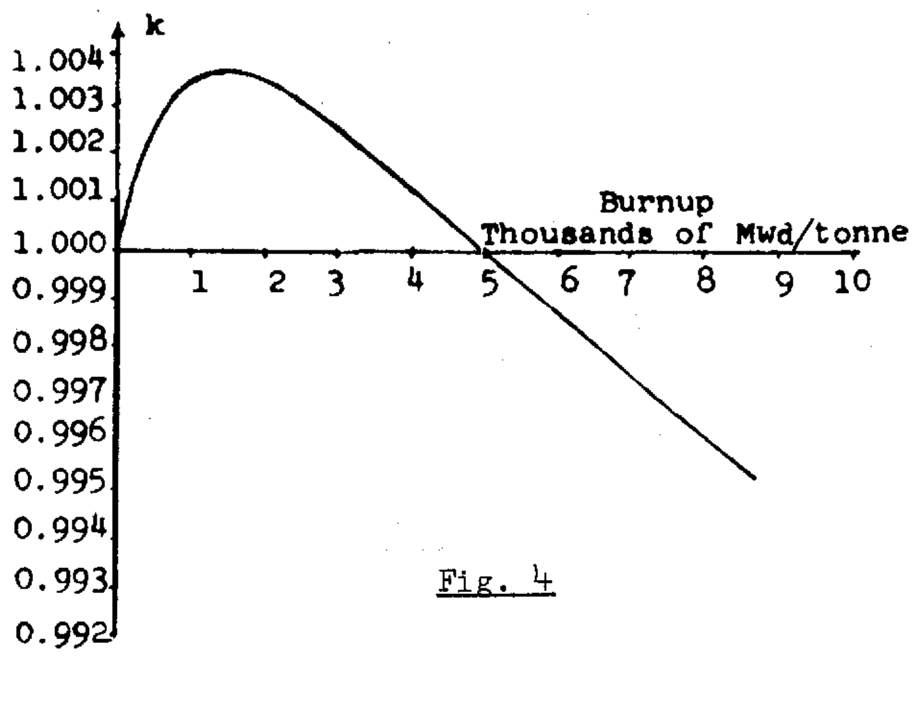

Fig. 4

ASSIGNMENT

1. Explain, in terms of the four factor formula, why a chain reaction can not be maintained by a reactor containing a homogeneous mixture of moderator and natural uranium.

2. What factors in the four factor formula are changed when the fuel is enriched and how do the changes enable criticality to be achieved?

3. How are the factors in the four factor formula changed when a heterogeneous system of moderator and fuel is used instead of a homogeneous system?

4. What advantages are to be gained by using fuel rods, in channels, rather than lumps of fuel arranged in the moderator?

5. (a) Give one definition of burnup.

   (b) What percentage of U-235 atoms are burnt if the burnup of natural uranium is 6500 Megawatt-days/tonne?

   (c) Why does the multiplication factor increase during initial fuel burnup and then decrease later?

A. Williams